

Singularities – Sapporo 1998

**ADVANCED STUDIES  
IN PURE MATHEMATICS 29**

Chief Editor: Eiichi Bannai (Kyushu University)

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**Singularities – Sapporo 1998**

Edited by

**Jean-Paul Brasselet** (Institut de Mathématiques de Luminy)

**Tatsuo Suwa** (Hokkaido University)

Published for the Mathematical Society of Japan by

KINOKUNIYA COMPANY LTD.

TOKYO, JAPAN

*This book was typeset by  $\mathcal{A}\mathcal{M}\mathcal{S}$ - $\mathcal{T}\mathcal{E}\mathcal{X}$  and  $\mathcal{A}\mathcal{M}\mathcal{S}$ - $\mathcal{L}\mathcal{A}\mathcal{T}\mathcal{E}\mathcal{X}$ , the  $\mathcal{T}\mathcal{E}\mathcal{X}$  macro systems of the American Mathematical Society, together with the style files `aspm.sty` and `aspmfm.sty` for  $\mathcal{A}\mathcal{M}\mathcal{S}$ - $\mathcal{T}\mathcal{E}\mathcal{X}$  written by Dr. Chiaki Tsukamoto and `aspmproc.sty` for  $\mathcal{A}\mathcal{M}\mathcal{S}$ - $\mathcal{L}\mathcal{A}\mathcal{T}\mathcal{E}\mathcal{X}$  written by Dr. Akihiro Munemasa.*

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*Edited by the Mathematical Society of Japan.*

*Published for the Mathematical Society of Japan and distributed in Japan by Kinokuniya Company Ltd., Tokyo, Japan.*

*Distributed outside Japan by the American Mathematical Society.*

*ISBN 4-314-10143-1*

*2000 Mathematics Subject Classification.*

*Primary 32Sxx; Secondary 14Cxx, 14Hxx, 14Jxx, 32Cxx, 57Rxx, 58Kxx*

## **Advanced Studies in Pure Mathematics 29**

**Chief Editor**

**Eiichi Bannai** (Kyushu University)

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PRINTED IN JAPAN

by Tokyo Shoseki Printing Co., Ltd.

## Preface

The present issue of *Advanced Studies in Pure Mathematics* constitutes the proceedings of the first of the two consecutive meetings held in Sapporo and Tokyo in July 1998. The idea of organizing an event on singularities and arrangements of hyperplanes came up in an informal discussion in 1996 of Jean-Paul Brasselet (IML, CNRS, Marseille), Mutsuo Oka (TMU, Tokyo), Kyoji Saito (RIMS, Kyoto), Tatsuo Suwa (Hokkaido University) and Hiroaki Terao (TMU, Tokyo). It was decided to have not one but two consecutive conferences in Sapporo and Tokyo, in order to express the relation between France and Japan in the domain of singularities on one hand and to commemorate the sixtieth birthyear of Peter Orlik on the other hand.

This project had been realized as the international symposium “Singularities in Geometry and Topology” and “Workshop on Mathematics Related to Arrangements of Hyperplanes”. The former is an expansion of the Franco-Japanese Seminar supported by CNRS and JSPS and was organized by J.-P. Brasselet and T. Suwa. The latter was organized by M. Oka and H. Terao with Michael Falk (Northern Arizona University) and Richard Randell (University of Iowa) as the program committee.

These two events have strong relationship. First of all, singularities and arrangements of hyperplanes are closely related from the mathematical viewpoint - the subjects studied and the tools used in the two theories are often the same; hypergeometric functions, singular de Rham complexes, resolution of singularities and Milnor fibration, to name a few. In fact, many mathematicians participated in the two conferences and some of them gave talks in both. Thus our original intention was to publish one joint volume as the proceedings of these two meetings. However, due to the page restriction, we had to publish in two separate volumes. We should mention that the proceedings of the other meeting will also be published in the series of *Advanced Studies in Pure Mathematics*.

The international symposium on “Singularities in Geometry and Topology” took place at Hokkaido University, Sapporo, July 6 - 10, 1998. There were sixty-eight participants, mostly from Japan and France but also from other countries including China, Mexico, Poland, Spain and USA. There were twenty-two lectures, whose main topics are singularities of curves, characteristic classes of singular varieties,  $\mathcal{D}$ -modules, motives, resolution of singularities, Milnor fibration, Enriques diagrams, Hilbert schemes and log-canonical singularities.

We briefly review the contents of the articles included in this volume. Singularities of curves are presented in various aspects: Mutsuo Oka gives properties of dual curves and dual singularities in particular geometry of cuspidal sextics and their dual curves. Antonio Campillo and Jorge Olivares study curves invariant by singular foliations on algebraic varieties and Poincaré-Hopf type formula, the paper of Hiro-o Tokunaga describes local types of singularities of plane curves and the topology of their complement.

Several papers are on the recent development about characteristic classes for singular varieties and, in particular, the so-called Milnor classes. It was remarked by José Seade and Tatsuo Suwa that, in the case of local complete intersections with isolated singularities, the difference between the Chern-Schwartz-MacPherson class and the virtual class is equal to the sum of Milnor numbers at the singular points. In the case of local complete intersections with arbitrary singularities, the difference is a homology class supported on the singular set and is called the Milnor class. The paper by Jean-Paul Brasselet gives a general history of characteristic classes for singular varieties, including a brief account of recent works of P. Aluffi, J.-P. Brasselet - D. Lehmann - J. Seade - T. Suwa, A. Parusiński - P. Pragacz and S. Yokura on Milnor classes. Paolo Aluffi gives an expression of Milnor classes for hypersurfaces in terms of Segre classes and defines the weighted Chern-Mather classes. Daniel Lehmann defines Milnor classes for local complete intersections in terms of Chern-Weil theory. Shoji Yokura, with Jean-Paul Brasselet, describe the behavior of Milnor classes in bivariant theory. Tatsuo Suwa defines characteristic classes of coherent sheaves on local complete intersections and gives a simple proof of the Riemann-Roch theorem for embeddings, which is used to compute these classes, in particular for the tangent sheaf. Isao Nakai proves the naturality of Stiefel-Whitney classes for singular maps using the Morin mappings.

Joël Briançon, Philippe Maisonobe and Michel Merle give a survey on characteristic varieties of  $\mathcal{D}$ -modules and singularities, in particular the relation with the conormal space and polar varieties. François Loeser justifies the use of motives to study the Milnor fiber, also the motivic Igusa integrals and the vanishing cycles. Shihoko Ishii studies quotients of isolated log-canonical singularities in detail. In particular, she determines all possible indices of such singularities in dimension three. The geometry of normal singularities is the theme of the Lê Dũng Tráng's paper, looking from various viewpoints: resolution of singularities, local link and so forth. Gérard Gonzalez-Sprinberg introduces the notions of

constellations of infinitely near points, toric constellations and toric Enriques diagrams. The paper of Vincent Blanloeil is devoted to a theory of cobordism for non-spherical links.

This international symposium was financially supported by the Centre National de la Recherche Scientifique (CNRS), the Japan Society for the Promotion of Science (JSPS), the Ministry of Education, Science, Sports and Culture of Japan, Sapporo International Communication Plaza Foundation, the Maison Franco-Japonaise in Tokyo and Plain Co., to whom we would like to express our deepest gratitude. We are also much indebted to Professors Tadao Oda and Eiichi Bannai for valuable advices in editorial process and to Professor Seiki Nishikawa for financial help in preparing this volume. Finally we would like to thank participants in the symposium and the referees of the articles for their cooperation.

Jean-Paul Brasselet  
Tatsuo Suwa

*All papers in this volume have been refereed and are in final form.  
No version of any of them will be submitted for publication elsewhere.*

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